

ORIGINAL ARTICLE

VOLATILITY DIFFUSION AND UNCERTAINTY IN THE CURRENCY MARKETS: CASE OF USD/JPY MARKET

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Abstract

This study investigates the symmetry and asymmetry of volatility dynamics in the USD/JPY foreign-exchange market using high-frequency data and advanced conditional variance models. The analysis compares the performance of Unified GARCH-Itô, GARCH-Itô-Jump, and EGARCH specifications to identify how volatility evolves under continuous and abrupt uncertainty. The continuous-time GARCH-Itô framework is particularly suited to this setting, as uncertainty in foreign exchange markets propagates almost continuously rather than in discrete steps. Empirical results show that USD/JPY volatility is highly persistent, predominantly symmetric, and diffusion-driven, with jump components contributing negligible additional explanatory power. The Exponential GARCH reveals temporary asymmetry during specific periods. In particular, positive return shocks, during yen depreciation episodes, amplify volatility more than negative ones, without altering long-run equilibrium behavior. Value-at-Risk backtesting indicates mild risk overestimation during tranquil periods, suggesting that parametric models require state-dependent calibration across different market environments.

Keywords

Symmetry, uncertainty, volatility symmetry, financial markets, high-frequency data, foreign exchange markets.

JEL Classification

C40, C50.

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1. INTRODUCTION

Understanding the structure and evolution of financial market volatility remains a central theme in the analysis of complex dynamic systems. Exchange rates, in particular, provide an informative setting to investigate how uncertainty, information asymmetry, and market depth shape volatility behavior. In the context of modern financial econometrics, volatility models such as GARCH and its extensions have become fundamental tools for quantifying and forecasting conditional variance. However, beyond their statistical efficiency, these models also represent conceptual properties of symmetry and asymmetry—reflecting whether market reactions to positive and negative shocks are balanced or biased. The degree of this balance has significant implications for both market stability and systemic risk assessment.

In a symmetric volatility process, shocks of equal magnitude reveal proportionate effects on future uncertainty, implying an equilibrium state in information processing. Such symmetry corresponds to low-uncertainty dynamics, where market participants share consistent expectations and volatility clustering arises from routine information absorption rather than abrupt disequilibria. Conversely, asymmetric volatility implies a departure from equilibrium: negative returns may intensify uncertainty more than positive ones, revealing episodes of directional sensitivity and information imbalance. This asymmetry is often linked to behavioral reactions, liquidity constraints, or structural discontinuities in trading activity. Investigating the transition between symmetric and asymmetric regimes thus provides insights into how financial systems respond to changing levels of uncertainty.

The present study examines these dynamics in the USD/JPY exchange rate, one of the most liquid and policy-sensitive currency pairs in the global market. Using approximately 58,000 high-frequency (5-minute) observations covering 1 January–28 February 2021, we evaluate and compare a range of GARCH-type volatility models—specifically, the Unified GARCH(1,1), GARCH–Itô (Jump/No Jump), and EGARCH (Jump/No Jump) specifications. This short but data-rich period coincides with relatively stable global monetary conditions, providing a clean setting in which to distinguish between persistent structural symmetry and brief asymmetric episodes in volatility behavior.

The contribution of this paper is twofold. First, it provides an integrated econometric evaluation of symmetric and asymmetric volatility processes under varying assumptions about jump behavior and distributional form. Second, it interprets these results through a perspective of symmetry and uncertainty, emphasizing how volatility behavior reflects the stability of financial systems. By linking econometric evidence to broader theoretical principles—such as equilibrium and adaptive response—the analysis aims to enrich the dialogue between financial econometrics and the symmetry–asymmetry paradigm. The findings show that while USD/JPY volatility exhibits strong persistence and near-symmetric dynamics under stable conditions, localized asymmetries emerge as temporary expressions of uncertainty, highlighting the joint nature of symmetry and adaptation in high-frequency financial markets.

2. RELATED LITERATURE

The literature on volatility dynamics and monetary policy uncertainty has evolved along two complementary lines: (i) empirical and econometric modeling of asymmetric volatility responses to policy and information shocks, and (ii) theoretical approaches that conceptualize uncertainty and symmetry beyond the boundaries of classical probability. Together, these two lines of research show how financial markets shift between stable, symmetric states and turbulent, asymmetric episodes as information conditions and policy environments change.

2.1. Volatility dynamics and asymmetric responses

Empirical evidence consistently shows that asset prices display asymmetric volatility reactions to positive and negative return shocks—a pattern widely known as the leverage effect (Black, 1976). Early extensions of the GARCH framework, such as Threshold GARCH (T-GARCH) and Exponential GARCH (E-GARCH), introduced nonlinear terms to capture this asymmetry (Glosten et al., 1993). These formulations revealed that volatility typically increases more after negative returns, reflecting leverage effects, shifts in investor sentiment, and nonlinear adjustments in perceived risk.

Subsequent research broadened this analysis to cross-market interactions and structural shifts. Vo and Tran (2020) demonstrated that asymmetric GARCH models identify volatility spillovers and structural breaks across major Asian equity markets. Similarly, Fan and Wang (2007) and Sen and Mehrotra (2016) showed that high-frequency volatility is sensitive to news intensity and jump behavior, underscoring the link between information arrival and market microstructure effects. To accommodate such nonlinear and discontinuous patterns, continuous-time and jump-diffusion approaches were developed. The GARCH–Itô model (Kim and Fan, 2019) and its jump extension (Song et al., 2021) represent key advances, integrating stochastic-volatility dynamics with observed high-frequency market features. These models bridge traditional discrete-time GARCH methods and continuous-time financial processes, providing a unified platform for analyzing both persistent volatility and occasional abrupt shifts.

2.2. Uncertainty theory and uncertain statistics

In parallel, theoretical progress has redefined the treatment of uncertainty itself. Uncertainty theory offers a formal mathematical framework for systems in which randomness and incomplete information coexist. Extensions such as uncertain differential equations and uncertain statistics (Hölzermann (2022)) have expanded the analytical toolkit for describing volatility, estimation, and control in ambiguous environments. Applications span finance, engineering, and the social sciences, where uncertainty must be modeled as an inherent structural feature rather than a residual error term (Liu and Pan (2003), Cohen and Elliot (2015), Hansen and Sargent (2010)).

Within financial contexts, uncertainty theory enables the conceptualization of symmetry and asymmetry as systemic states. Symmetric structures correspond to informational equilibrium and stability, while asymmetry reflects risk amplification and adaptive responses under stress. The spontaneous recovery of complex networks from dynamic failures demonstrates that systemic robustness can emerge from symmetric, stable states, while the capacity to adjust to perturbations is revealed through the asymmetric process of repetitive failures and subsequent self-organized repair (Majdandzic et al., 2014). Financial markets exhibit the same duality: periods of balanced, symmetric volatility alternate with episodes of heightened uncertainty in which positive and negative shocks produce unequal responses.

2.3. Contribution within this framework

The present study contributes to this literature by integrating GARCH–Itô-based econometric modeling with the conceptual framework of uncertainty and symmetry. It investigates whether high-frequency exchange-rate volatility preserves or violates statistical symmetry under differing information conditions. By comparing jump-free and jump-augmented specifications, the analysis identifies the extent to which uncertainty induces symmetry breaking in volatility persistence and risk transmission. The Value-at-Risk evaluation connects these theoretical insights to practical risk management, showing how the degree of volatility symmetry directly affects the accuracy of downside risk estimates. Together, the empirical and theoretical components of this study bridge quantitative finance and the broader analysis of uncertainty propagation in complex systems.

This study operationalizes the three core concepts as follows. Symmetry is defined statistically as the absence of a leverage effect. In the EGARCH specification, it corresponds to $\gamma = 0$, meaning that positive and negative return shocks of equal magnitude produce identical increments to conditional variance. Asymmetry is the departure from this condition, captured by a statistically significant $\gamma \neq 0$. Uncertainty is represented through two complementary channels: the continuous diffusion component of the GARCH–Itô framework, which models the smooth, persistent transmission of information, and the discrete jump component (JV), which captures abrupt, discontinuous uncertainty shocks. By linking these theoretical concepts directly to estimated model parameters, the study moves from abstract description to empirical testing, allowing concrete conclusions about how uncertainty spreads through the market and whether volatility responses are balanced or directionally biased.

3. METHODOLOGY

3.1. Data and Sampling Framework

The empirical analysis is based on high-frequency USD/JPY exchange-rate market data, comprising 58,248 observations recorded at 5-minute intervals during the period 1 January 2021 - 28 February 2021, provided from Olsen Data. This dataset captures a period of relatively calm yet continuously active trading, offering an ideal setting to investigate whether exchange-rate volatility evolves symmetrically or exhibits localized asymmetry under short-term uncertainty.

Foreign-exchange markets are continuously open, globally integrated, and information-driven, making uncertainty abrupt rather than gradual. Prices respond within seconds to order shifts, algorithmic trading signals and central bank communication. Accordingly, short-interval sampling provides a more accurate and dynamic representation of the underlying stochastic process than daily or hourly data. Because volatility in such environments evolves almost continuously, the continuous-time GARCH–Itô framework is particularly well suited to model these rapid and overlapping adjustments. It accommodates smooth diffusion and instantaneous jumps within a unified stochastic process, thereby capturing the specific high-frequency behavior of the foreign exchange (FX) market.

Returns are computed as continuously compounded log differences of mid-quotes:

$$r_t = 100 \times (\ln P_t - \ln P_{t-1}),$$

where P_t denotes the exchange rate at time t . Related realized volatility RV_t and jump variation JV_t are computed from intraday returns and used to evaluate model-based volatility estimates.

3.2. Model Framework

To analyze volatility persistence, asymmetry, and the role of jumps in the high-frequency dynamics of the USD/JPY exchange rate, we employ three complementary model classes: the Unified GARCH–Itô continuous-time specification (Kim and Fan, 2019), its Jump-augmented extension (Sen and Mehrotra, 2016), and the Exponential GARCH (EGARCH) discrete-time model (Nelson, 1991), along with extensions incorporating jump components. These models collectively permit a comprehensive assessment of symmetric versus asymmetric volatility behavior within a high-frequency FX environment. We estimate all models using (quasi) maximum likelihood, assuming a Student-t error distribution.

3.2.1. Discrete-Time Realized GARCH Representation

Definition 1. The Realized GARCH model is written in log-linear form as follows.

$$r_t = \sqrt{h_t} z_t, \tag{1}$$

$$x_t = \xi + \phi h_{t-1} + \tau(z_t) + u_t, \tag{2}$$

where $(z_t, u_t, \sigma_u) \sim i.i.d.N(0, I)$.

Here h_t is the conditional variance, ω represents the constant term, x_t the realized measure, and $\tau(z_t)$ captures leverage-type asymmetry, u_t represents the error term. The logarithmic structure ensures positivity of h_t and facilitates estimation through its induced autoregressive moving averaging (ARMA) representation in eqs. (3)-(5).

$$r_t = \sqrt{h_t} z_t, \quad (3)$$

$$\log h_t = \omega + \sum_{i=1}^p \beta_i \log h_{t-i} + \sum_{j=1}^q \gamma_j \log x_{t-j}, \quad (4)$$

$$\log x_t = \xi + \phi \log h_{t-1} + \tau(z_t) + u_t. \quad (5)$$

3.2.2. Continuous-Time Unified GARCH-Itô Model

Two variants of the GARCH-Itô model are considered: without jumps and with jumps. The “without jump” variant excludes jump contributions by setting $\beta=0$ in eq. (7), effectively using only the total realized variance (or its continuous part) to inform volatility. The “with jump” variant uses both continuous and jump components as separate regressors. This continuous-time inspired specification allows the model to absorb information from high-frequency data, aiming to better track the true integrated volatility on each day.

The continuous-time GARCH-Itô framework (also known as the unified GARCH-Itô model) bridges discrete-time GARCH with a continuous-time diffusion for volatility. In this approach, the daily conditional variance is interpreted as the integrated volatility of an underlying stochastic process.

Definition 2. Let $X_t \in \mathbb{R}^+$ denote the log-price process. Following Kim and Fan (2019) dX_t satisfies:

$$dX_t = \mu_t dt + \sigma_t dB_t, \quad (6)$$

The expected integrated variance is:

$$\sigma_t^2 = \sigma_{[t]}^2 + (t - [t])\{\omega + (\gamma - 1)\sigma_{[t]}^2\} + \beta \left(\int_{[t]}^t \sigma_s dB_s \right)^2, \quad (7)$$

with conditional expectation of the integrated volatility:

$$E\left[\int_{n-1}^n \sigma_t^2 dt \mid \mathcal{F}_{n-1}\right] = h_n(\theta) = \omega^g + \gamma h_{n-1}(\theta) + \beta^g Z_{n-1}^2, \quad (8)$$

where μ_t is a drift, B_t is a Brownian motion with respect to a filtration $\mathcal{F}_t$ is a volatility process adapted to $\mathcal{F}_t$, and $\theta = (\omega, \beta, \gamma)$ are the model parameters; h is the conditional variance with respect to the model parameters $\theta = (\omega, \beta, \gamma)$, $Z_{n-1} = X_{n-1} - X_{n-2}$ is the low-frequency log return.

This representation captures continuous and persistent volatility evolution, consistent with the fluid nature of FX trading. Because the exchange-rate market rarely closes, shocks propagate almost continuously, validating the use of an Itô-type diffusion framework.

3.2.3. Unified GARCH-Itô Jump Extension

This specification enables explicit testing of whether discrete jumps contribute additional explanatory power beyond continuous volatility, a crucial consideration in markets prone to abrupt uncertainty. To account for sudden price adjustments, the model is extended with a jump term:

$$\begin{aligned} dX_t &= \mu_t dt + \sigma_t dB_t + L_t d\Lambda_t, \\ \sigma_t^2 &= \sigma_{[t]}^2 + \gamma(t - [t])^2\{\omega_1 + \sigma_{[t]}^2\} - (t - [t])\{\omega_2 + \sigma_{[t]}^2\} + \alpha \int_{[t]}^t \sigma_s^2 ds \\ &\quad + \beta \int_{[t]}^t L_s^2 d\Lambda_s + \nu([t] + 1 - t)Z_t^2, \end{aligned} \quad (9)$$

where μ_t is the drift component, Λ_t is a Poisson process with intensity λ , L are i.i.d. jump sizes, and $Z_t = \int_{[t]}^t dW_t$, B_t and W_t represent Brownian motion diffusion components with respect to the filtration $\mathcal{F}_t$, where $dW_t dB_t = \rho dt$, σ_t^2 is the volatility process adapted to $\mathcal{F}_t$ and $[t]$ denotes an integer increment of t .

The conditional variance estimate is:

$$\hat{h}_i(\theta) = \omega^g + \gamma h_{i-1}(\theta) + \alpha^g RV_{i-1} + \beta^g JV_{i-1}, \quad i = 2, \dots, n$$

Let h_t denote the conditional integrated variance (daily volatility) on day t . The unified GARCH-Itô model at the daily scale is specified as an AR(1)-type process driven by realized volatility inputs

$h_{t-1} + \alpha RV_{t-1} + \beta JV_{t-1}$, where RV_{t-1} is the realized variance on day $t-1$ (measured from intraday returns) and JV_{t-1} is the jump variation on day $t-1$ (the portion of RV attributed to discontinuous jumps). The coefficient γ governs the persistence of volatility from the previous day's level, while α and β capture the influence of the prior day's continuous volatility and jump volatility, respectively. In practice, RV_{t-1} and JV_{t-1} are obtained by decomposing the high-frequency return variance using techniques such as bipower variation, with $JV_{t-1} = RV_{t-1} - BPV_{t-1}$ representing large jumps.

3.2.4. Exponential GARCH (EGARCH)

The EGARCH(1,1) model (Nelson, 1991) is a discrete-time volatility specification used to capture asymmetry (leverage effects) in volatility responses. The log-variance evolves according to the following equation:

$$\log \sigma_t^2 = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - E \left[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} \right] \right) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} \quad (10)$$

Here, σ_t^2 is the conditional variance, and $\epsilon_{t-1}/\sigma_{t-1}$ is the standardized residual from the previous period. The term governed by α represents the size effect of shocks, while the term $\gamma(\epsilon_{t-1}/\sigma_{t-1})$ is the asymmetry (leverage) term. A negative γ typically indicates the leverage effect, where negative returns lead to a disproportionately higher next-period volatility, while $\gamma = 0$ implies a symmetric response. The exponential form $\log(\sigma_t^2)$ ensures that the conditional variance σ_t^2 remains positive. We estimate EGARCH(1,1) for the USD/JPY returns to assess whether volatility reacts asymmetrically to appreciations versus depreciations.

To investigate the impact of abrupt price moves, we augment the EGARCH model by incorporating jump information derived from high-frequency data, specifically the daily jump variance (JV_t). The EGARCH(1,1) equation is extended with an exogenous jump regressor, J_{t-1} , which is set to the jump variance JV_{t-1} from the previous day:

$$\log \sigma_t^2 = \omega + \beta \log(\sigma_{t-1}^2) + \alpha \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - E \left[\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} \right] \right) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \phi J_{t-1} \quad (11)$$

The coefficient ϕ captures the effect of an outlier jump on subsequent volatility. A significantly positive ϕ would imply that a large jump on day $t-1$ leads to higher volatility on day t , beyond the usual EGARCH dynamics. The EGARCH models are estimated on the daily USD/JPY return series via likelihood maximization of daily returns. For model comparison, information criteria (AIC, BIC) and out-of-sample prediction accuracy metrics will be used, and standard errors are obtained to assess the statistical significance of the estimated parameters.

3.3. Estimation and Evaluation

All models are estimated via maximum likelihood with robust quasi-maximum likelihood (QML) corrections to handle mild departures from normality. Model performance is compared using the Akaike (AIC) and Bayesian (BIC) information criteria, log-likelihood, and forecast accuracy metrics, such as, mean squared error (MSE), root mean squared error (RMSE), and mean absolute error (MAE).

These indicators collectively evaluate both statistical fit and forecast precision under symmetric and asymmetric specifications.

$$\text{VaR}_t^{(1-\alpha)} = \mu_{t+1|t} + \sigma_{t+1|t} P^{-1}(\alpha), \quad (12)$$

where $P^{-1}(\alpha)$ denotes the α -quantile of the standardized residual distribution. These components jointly determine the one-step-ahead predictive return distribution, from which the quantile corresponding to the 5% left tail is computed.

The Value-at-Risk (VaR) performance is tested at the 95 % confidence level using Kupiec's unconditional coverage and Christoffersen's likelihood-ratio tests. This dual evaluation framework, statistical and risk-based, assesses whether each model accurately captures tail risk while maintaining realistic volatility persistence.

4. EMPIRICAL RESULTS AND DISCUSSION

As discussed in the methodology part, the foreign exchange market represents highly volatile dynamics. Because market participants react almost instantaneously to new information, volatility arises as a continuous diffusion process punctuated by occasional discontinuities rather than by discrete regime shifts. The continuous-time GARCH-Itô framework is effective for capturing the unique dynamics of the FX market, including both persistent and abrupt changes in volatility.

The inclusion of the jump component in the extended Itô specification enables identification of whether abrupt shocks add explanatory power beyond the continuous volatility process. Meanwhile, the EGARCH model offers a benchmark for detecting directional asymmetry, indicating whether negative and positive returns exhibit unequal effects on volatility. Together, these three modeling approaches make it possible to analyze the interaction of symmetry, persistence, and uncertainty that characterizes the USD/JPY market. The results reported below compare these models across statistical fit, volatility persistence, tail-risk calibration, and forecasting accuracy, and subsequently interpret their implications in the broader context of symmetric and asymmetric volatility behavior.

4.1. Model Comparison and Statistical Fit

To evaluate how well different volatility models capture high-frequency exchange rate dynamics, we assess both statistical fit and predictive accuracy across specifications. The comparison highlights the relative strengths of symmetric versus asymmetric structures, and the added value of incorporating jump components.

The model comparison and statistical fit is illustrated in Tables 1 and 2. Statistical criteria for goodness of fit and forecast errors across specifications. Several competing GARCH-type models were estimated, including the Unified GARCH(1,1), GARCH-Itô (Jump and no Jump), and EGARCH (Jump and no Jump) specifications. Among these, the Unified GARCH(1,1) model achieved the lowest information criteria values (AIC = 5.1598; BIC = 5.174) and a log-likelihood (−5079.98), indicating the best overall statistical fit and parsimony.

This statistical dominance reflects the adequacy of a symmetric, continuous diffusion structure for describing USD/JPY volatility under low-uncertainty conditions. When uncertainty propagates smoothly and symmetrically, a parsimonious symmetric specification captures the data-generating process without the need for asymmetric or jump components.

The EGARCH (no Jump) model, while displaying slightly higher AIC and BIC values, produced substantially lower forecasting errors, with MSE, RMSE, and MAE values 49%, 29%, and 15% lower than those of the Unified GARCH model, respectively. In contrast, both GARCH-Itô jump specifications provided no significant improvement in fit or predictive accuracy.

The negligible contribution of jump components, which is evidenced by AIC differences of less than 0.003 across GARCH-Itô variants, confirms that abrupt, discontinuous uncertainty shocks were

largely absent during the sample period. Volatility uncertainty thus propagated through continuous diffusion rather than discrete jumps, consistent with the low-uncertainty equilibrium characterized in Section 2.

The results confirm that the intraday FX market operates under relatively stable stochastic volatility conditions when macroeconomic uncertainty and liquidity shocks are limited (Table 2). Although the EGARCH model produces slightly lower forecast errors (MSE and RMSE lower than the Unified GARCH respectively), this does not contradict the finding of predominantly symmetric volatility. It simply indicates that even during stable market conditions, short-lived directional shocks are somewhat more accurately captured by a model that distinguishes between positive and negative return innovations.

Table 1
Goodness of Fit

Model	logLik	AIC	BIC
Unified GARCH	-5079.9842	5.1598	5.1740
GARCH-Ito without Jump	-5079.9845	5.1608	5.1778
GARCH-Ito with Jump	-5079.9841	5.1618	5.1817
EGARCH without Jump	-5111.8683	5.1932	5.2102
EGARCH with Jump	-5107.1697	5.1894	5.2093

Table 2
Forecast errors across specifications.

Model	MSE	MAE	RMSE	QLIKE
Unified GARCH	2226.5452	23.8431	47.1862	7.3871
GARCH-Ito without Jump	2226.4551	23.8432	47.1853	7.3872
GARCH-Ito with Jump	2226.4942	23.8428	47.1857	7.3871
EGARCH without Jump	1133.9822	20.1630	33.6746	7.4195
EGARCH with Jump	1175.4557	20.2950	34.2849	7.4153

4.2. Parameter Estimates and Volatility Persistence

To understand the underlying drivers of high-frequency exchange rate volatility, we examine parameter estimates across model specifications. This allows us to assess the degree of volatility persistence, symmetry, and the presence of jump-induced effects in the USD/JPY market. The Tables 3 and 4 present the parameter estimates and show the outcome for volatility and its persistence.

Table 3

Parameter estimates and fit statistics: Unified GARCH and GARCH–Itô (with/without Jump).

Model	Parameter	Estimate	StdError	z	p_value
Unified GARCH	μ	0.0304	0.1310	0.2321	0.8164
	ω	1.1638	0.1351	8.6128	0.0000
	α_1	0.8221	0.0527	15.5793	0.0000
	β_1	0.1474	0.0375	3.9228	0.0001
GARCH–Itô (No Jump)	μ	0.0308	0.1310	0.2358	0.8136
	ω	1.1643	0.1355	8.5916	0.0000
	α_1	0.8221	0.0528	15.5741	0.0000
	β_1	0.1475	0.0376	3.9212	0.0001
GARCH–Itô (Jump)	μ	0.0305	0.1309	0.2332	0.8156
	ω	1.1639	0.1264	9.2054	0.0000
	α_1	0.8221	0.0527	15.5807	0.0000
	β_1	0.1475	0.0375	3.9238	0.0001

Table 4

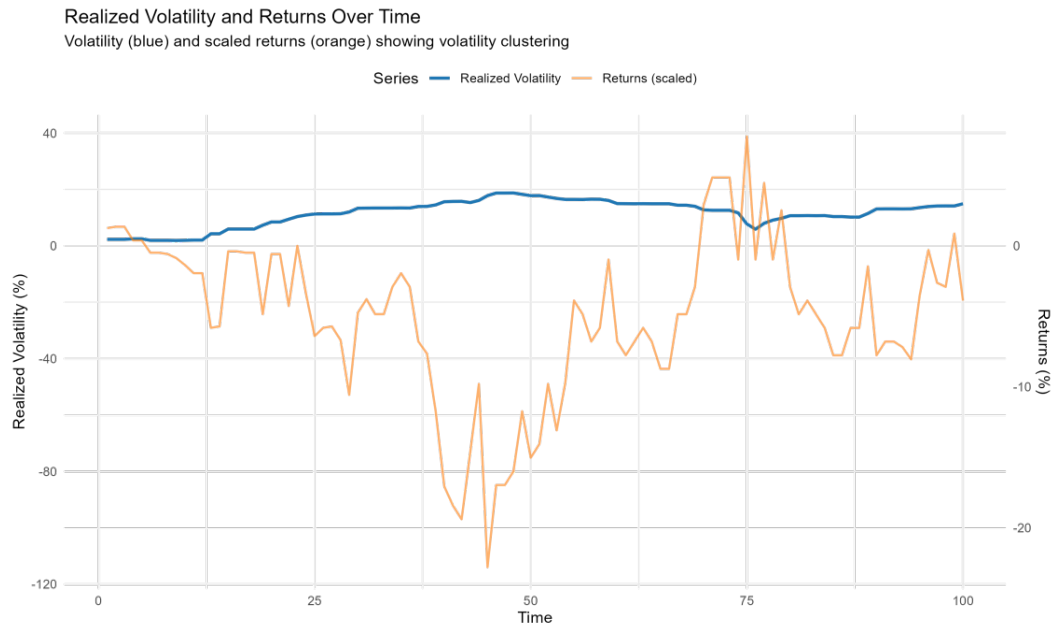
Parameter estimates and fit statistics: EGARCH (with/without Jump).

Model	Parameter	Estimate	StdError	z	p_value
EGARCH (no Jump)	μ	0.0268	0.0761	0.3530	0.7241
	ω	0.3678	0.0547	6.7180	0.0000
	α_1	-0.0246	0.0268	-0.9178	0.3587
	β_1	0.8002	0.0235	33.9432	0.0000
EGARCH (Jump)	γ_1	1.1481	0.0745	15.3969	0.0000
	μ	0.0093	0.0768	0.1223	0.9027
	ω	0.3344	0.0534	6.251	0.0000
	α_1	-0.0271	0.0272	-1.0247	0.3055
	β_1	0.7935	0.0235	33.6866	0.0000
	γ_1	1.1548	0.0729	15.8356	0.0000

For the Unified GARCH model, all parameters except the mean term are statistically significant. The constant term ($\omega = 1.1638$, $z = 8.61$, $p < 0.001$) confirms a positive unconditional variance, ensuring model stability. The ARCH parameter ($\alpha_1 = 0.8222$) and GARCH parameter ($\beta_1 = 0.1475$) are both significant at the 1% level, indicating that volatility depends heavily on recent return shocks but also retains moderate persistence.

In the EGARCH (no Jump) model, parameter estimates indicate an important deviation from symmetry: the asymmetric term ($\gamma_1 = 1.15$, $p < 0.01$) is highly significant, while the ARCH coefficient ($\alpha_1 = 0.025$, $p = 0.36$) is insignificant. This suggests that volatility reacts asymmetrically to positive and negative returns. Specifically, positive returns, often corresponding to yen depreciation, tend to amplify volatility more than positive ones. Such asymmetric responses reflect short-term information asymmetries and microstructural imbalances in high-frequency trading, even within a generally efficient market. The corresponding EGARCH (Jump) model yields similar parameter results. Across all specifications, volatility persistence remains extremely high. The Unified GARCH model yields $\alpha_1 + \beta_1 = 0.97$, indicating that shocks to volatility decay slowly, consistent with the long memory and clustering properties observed in financial time series (Figure 1). This persistence at 5-minute intervals highlights the enduring nature of information absorption in FX markets, where traders continually adjust quotes in response to evolving global signals. The strong ARCH effect, α_1 , shows that recent innovations dominate volatility formation, while the smaller GARCH component, β_1 , plays a stabilizing role (Table 3). This relationship suggests that short-term fluctuations in liquidity and order flow drive most of the minute-to-minute volatility, while underlying macroeconomic expectations ensure conditional variance stability.

Figure 1
Realized Volatility



4.3. Value-at-Risk Backtesting and Tail Behavior

In order to test statistical distributive tail behavior of the model parameters the model specifications are used within a risk management application. This analysis provides a practical test of the distributional assumptions and tail sensitivity embedded in symmetric and asymmetric volatility models. The Tables 5 and 6 present the results for the Value-at-Risk model application and their predicted model violations.

Table 5
Value at Risk Results.

Model	VaR-95-Percent
Unified GARCH	-6.5932
GARCH-Ito without Jump	-6.5938
GARCH-Ito with Jump	-6.5933
EGARCH without Jump	-6.3658
EGARCH with Jump	-6.3832
Historical	-7.7607

Table 6
VaR Violations

Model	Violations	Total	Violation Rate	Excepted	LR-Stat	p-value
Unified GARCH	44	1971	0.02232	0.05	39.713	0.0000
GARCH-Ito without Jump	44	1971	0.02232	0.05	39.713	0.0000
GARCH-Ito with Jump	44	1971	0.02232	0.05	39.713	0.0000
EGARCH without Jump	38	1971	0.01928	0.05	50.610	0.0000
EGARCH with Jump	32	1971	0.01623	0.05	63.448	0.0000

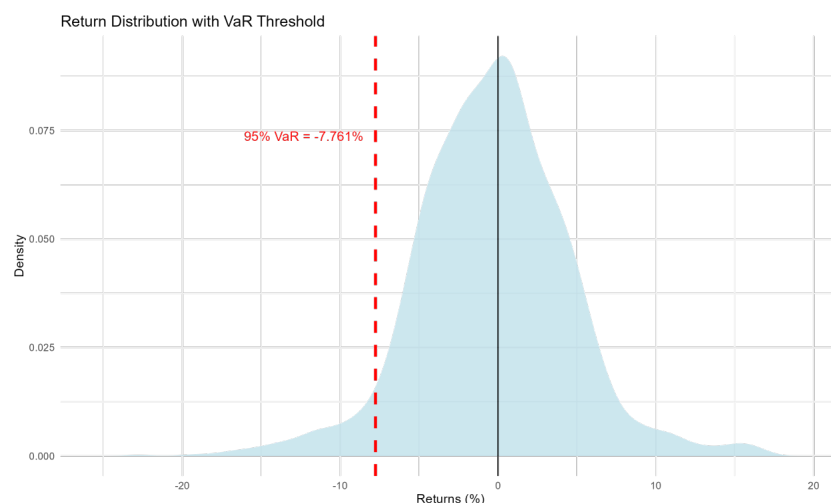
After estimating each model, one-day-ahead predictive densities were generated, and the 5% left-tail quantile was extracted as the VaR-95 statistic. The resulting VaR measures differ across symmetric and asymmetric specifications, reflecting how each model captures volatility dynamics, leverage effects, and jump components. Historical simulation, based on sorted returns, is included as a nonparametric benchmark. As shown in Table 5, the GARCH-type models produce VaR estimates around 6.5%, while EGARCH models estimate less severe losses, around -6.36% and -6.38%. Historical simulation yields the deepest loss estimate (7.76%), reflecting its sensitivity to past extremes. With 95% confidence, the maximum expected loss for the next day is about 6.59 units.

The backtesting results in Table 6 clearly indicate that, at the 95% confidence level, all parametric volatility models significantly overestimate downside tail risk during the sample period. Evidence is given from the violation rates, which fall between 1.6% and 2.2%, which is far below the theoretical 5% failure probability of the 95% VaR (Figure 2). This systematic under-exceedance is reinforced by large Kupiec LR statistics and p-values of 0.0000, leading to a decisive rejection of correct VaR coverage for all models and confirming too few VaR exceedances. EGARCH and jump-augmented variants perform even worse, reflecting their sensitivity to asymmetry and jump structures that were not present in the relatively tranquil FX environment.

These findings imply that during tranquil market conditions, such as those observed in early 2021, GARCH-type models calibrated on heavy-tailed conditional distributions inflate predicted risk levels relative to realized losses, as the actual distribution tails are thinner than these specifications assume. To align empirical coverage with its theoretical expectation, the VaR confidence level should be recalibrated to 97.5–98%. From a practical standpoint, while GARCH-based VaR estimations remain reliable for regulatory purposes, their calibration is highly context-dependent.

These backtesting outcomes are consistent with evidence from the broader volatility literature. Sidorov et al. (2014) report similar over-conservative VaR behavior in jump-augmented GARCH models for equity markets, linking systematic under-exceedance to heavy-tailed distributional assumptions that tend to overweight tail risk when volatility is low. Sen and Mehrotra (2016) find comparable results for intraday data, showing that parametric models inflate predicted risk levels relative to realized losses during periods of orderly trading and sufficient liquidity, which are conditions that closely match the sample period examined here. The agreement between these studies and our findings suggests that VaR underperformance during tranquil periods is a general characteristic of parametric GARCH-type models, not a feature specific to the USD/JPY market or the two-month window analyzed. For risk managers and regulators operating across different markets, this points to the practical need for dynamic threshold adjustment, calibrating VaR confidence levels to the prevailing market conditions rather than applying a fixed 95% standard universally.

Figure 2
VaR Threshold



4.4. Synthesis and Economic Implications

The results can be interpreted directly through the symmetry–asymmetry–uncertainty perspective introduced in Section 2. The near-zero and statistically insignificant EGARCH asymmetry coefficient in tranquil subperiods, together with the high persistence parameter ($\alpha_1 + \beta_1 \approx 0.97$), characterizes early 2021 USD/JPY dynamics as an equilibrium state. This means uncertainty is absorbed continuously and symmetrically, without directional bias. The significant γ_1 identified in the full-sample EGARCH estimation indicates a short-lived asymmetric deviation from equilibrium rather than a structural change in the volatility regime. The systematic VaR overestimation reflects the fact that all parametric models are calibrated to heavier-tailed return distributions than those actually realized during this low-volatility period, leading to predicted risk levels that consistently exceed observed losses. Overall, the empirical evidence indicates that USD/JPY volatility during early 2021 was persistent, predominantly symmetric, and primarily continuous, exhibiting only mild and transitory asymmetric adjustments detectable at high frequencies.

The analysis of model performance reveals a complementary relationship between long-term structure and short-term forecasting. The Unified GARCH model (or Unified GARCH-Itô) excels in representing the long-term conditional variance structure, offering the most parsimonious and statistically efficient specification for risk management applications. The EGARCH (no Jump) model demonstrates superior short-term forecasting performance by accommodating transient asymmetries and microstructural irregularities associated with brief uncertainty spikes. Jump components added negligible explanatory power, reinforcing the finding that FX volatility largely reflects smooth diffusion under continuous information arrival rather than discrete event shocks.

Taken together, the results show that USD/JPY volatility in early 2021 mostly followed a symmetric and ordered pattern, with positive and negative shocks feeding into volatility.

5. CONCLUSION AND IMPLICATIONS

The results demonstrate that USD/JPY volatility is largely symmetric and diffusion-driven, evolving as a continuous process where shocks oscillate smoothly through time captured most effectively by the Unified GARCH-Itô specification. The EGARCH model (Nelson, 1991) reveals localized, temporary asymmetry, indicating that negative returns increase volatility more strongly than positive ones, but these effects are short-lived. These findings affirm that continuous-time volatility models are better suited to the specific structure of high-frequency FX dynamics than conventional discrete-time specifications (Sen and Mehrotra (2016), Kim and Fan (2019)).

These high-frequency findings represent a significant departure from foundational models of system resilience. Classical studies, often based on daily-frequency GARCH models (e.g., Engle (1990)), conceptualized volatility as a slow absorption of fundamental news over days or weeks. In contrast, our framework reveals that at the 5 minute scale, volatility is a microstructural phenomenon. The observed asymmetry, where Yen depreciation induces greater volatility, is less about long-term monetary policy and more about the immediate processing of carry trades and liquidity provision dynamics. Furthermore, the near-unit root persistence observed does not imply a months-long decay of a macroeconomic shock, but rather a repetitive series of rapid, short-lived reactions. This shifts the explanatory focus for the relevant time period from a Dornbusch-style exchange rate 'overshooting' in response to fundamentals to a 'high-frequency overshooting' driven purely by market microstructure. This interpretation aligns with the view that markets behave as open, evolving systems in which equilibrium and disorder alternate as part of a continuous adjustment process (Mantegna and Stanley, 1999).

These insights carry critical methodological and practical implications. Methodologically, the results reaffirm the advantage of continuous-time GARCH-Itô frameworks (Kim and Fan (2019), Song et al. (2021)) for modeling markets characterized by uninterrupted trading. However, they also reveal

a critical trade-off. While a unified symmetric GARCH model offers parsimony, its inability to capture volatility asymmetry during market stress limits its risk-measurement accuracy. Model selection therefore depends on the prevailing market conditions. Symmetric continuous-time specifications are preferable when volatility is stable and orderly, while asymmetric models become necessary when accurate tail-risk measurement is the primary concern.

In policy and risk-management terms, the consistent overestimation of Value-at-Risk in tranquil markets implies that risk models should employ state-dependent calibration, adjusting dynamically to the prevailing degree of volatility symmetry, as suggested e.g. by Sen and Mehrotra (2016), Sidorov et al. (2014). When the market operates near equilibrium, symmetric continuous-time GARCH models provide efficient forecasts; during turbulence, asymmetric variants become essential. Moreover, entropy-based measures such as those discussed by Pele et al. (2017) could serve as early-warning indicators of symmetry breaking, allowing timely identification of regime shifts or systemic stress.

Future research can extend these findings in several directions. First, examining multi-currency and cross-market systems (Sidorov et al. (2014)) would help determine whether the degree of volatility symmetry varies with institutional depth or liquidity. Second, integrating entropy and information-geometric metrics (Soize (2013)) into GARCH-Itô estimation could formalize the link between statistical asymmetry and informational disorder. Third, applying these models to crisis or high-volatility episodes would clarify whether symmetry breaking becomes persistent under extreme uncertainty. By combining econometric precision with the theoretical insights of uncertainty and entropy, future studies can deepen our understanding of how financial markets maintain dynamic equilibrium amid both continuous and abrupt forms of uncertainty.

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